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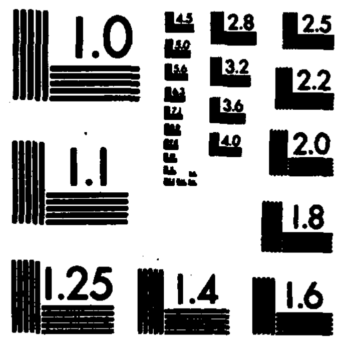

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A NUMERICAL ANALYSIS OF SAMPLING PLANS BASED ON
PRIOR DISTRIBUTION AND COSTS

by

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Summary

This paper deals with some pitfalls linked with the sampling model based on prior distribution and costs. First, a model is designed which encompasses most of the existing Bayesian cost models. The efficiency of sampling plans is investigated in a numerical study. It is shown that under realistic assumptions, described by Dodge (1969) and Schilling (1982), sampling plans based on prior distributions and costs are only efficient in an outlier model, i.e. if almost all lots are of good quality and only a low number of lots, denoted as outlier lots, have very poor quality. Furthermore, it is demonstrated that for the Polya distribution a gain of sampling is linked with a high percetage of rejections, i.e. when the prior distribution cost relationship is such that less than 5% of the lots should be rejected sampling becomes inefficient.

1. Introduction

Attributes acceptance sampling plans are used throughout the world. Sets of single, double and multiple sampling plans as well as sequential sampling plans have been devised by different organizations. Most often they are risk-based sampling plans, i.e. the plans are selected according to certain types of consumer and producer risks as depicted by the operating characteristic (OC). The majority of practitioners today are applying plans such as MIL-STD 105D because they are widely available and widely accepted. These procedures have been criticized since they are not based on economical criteria such as cost of inspection, cost of passing defectives and cost of rejecting good products. Furthermore, these plans do not explicitly take into consideration the past history of similar lots submitted previously for inspection purposes.

During the past thirty years a number of papers have been written dealing with a Bayesian approach based on a cost criterion. This approach to sampling plan design takes into account the past lot quality history. Bayesian sampling plans require an explicit specification of the distribution of defectives from lot to lot; this distribution is denoted as the prior distribution.

A summary of the theoretical foundation for selecting single sampling plans on the basis of a prior distribution and costs is given by Hald (1960), who also considers double sampling plans where the size of the second-stage sample is decided before the first sample is taken. Only the decision of whether to take a second sample is dependent upon the results of the first-stage sample. Pfanzagl (1963) investigates double sampling plans where the sample size and the acceptance number of the sample at the second stage depend on the outcome of the first sample. He also compares the costs of a double sampling plan with those of a single sampling plan. Pfanzagl and Schüler (1970) investigate the efficiency of sequential sampling plans based on prior distribution and costs. Multistage sampling procedures based on prior distributions and costs are discussed by Schüler (1967). The results are only of theoretical interest. Wortham and Wilson (1977) have considered a backward recursive technique for optimal sequential sampling plans.

In spite of the considerable number of papers concerning this subject, there is still a gap between the theoretical work and the application of cost optimal Bayesian sampling plans in industries, see papers by Hoadley, (1981), Hoadley and Buswell (1983) and Hoadley (1981). This paper considers some pitfalls linked with the application of a sampling model based on prior distribution and costs. Although this approach naturally takes into account economic factors and thus seems to be superior to a risk-based sampling plan, we shall see that it does not offer much economic gain in the majority of situations, with the exception of the outlier model. It shall be shown that the general model is based on such specific assumptions that in many real cases acceptance without inspection would be the best way to handle the lots. In order to show this, we will first present a cost model which encompasses most of the known models based on prior distributions and costs such as those discussed in Hald (1960), Wetherill (1960) Pfanzagl (1963) and Wortham (1971). The model discussed here allows not only the determination of the optimal sample size n and the acceptance under c but also the optimal stage of a sampling plan, i.e. we will determine whether a single, double, multistage or sequential sampling plan is optimal.

Then a numerical study will reveal that under cost assumptions and prior distributions which ensure that more than 95% of the lots are acceptable, none of these sampling plans will be superior to acceptance without inspection, unless our aim is to detect outlier lots with a particularly high percentage of defectives. The conclusion offers some explanations for the weaknesses of the models based on prior distribution and costs.

2. The model

2.1 The costs

Most of the papers concerning sampling plans based on prior distributions and costs begin by assuming that costs for guarantee and repair as well as marginal costs for sampling and rejecting an item are given. Their magnitude in numerical examples is chosen rather arbitrarily due to the priciple difficulties in assigning these costs. On order to avoid undue consideration of impractical cases let us consider the main economic factors which are involved in practice and discuss our ability to measure them. The main factors are inspecting lots, rejecting lots, passing defective items. We do not consider extra costs for repairing defective items since all of the defective items have to be repaired either during the stage of sampling or when they are detected by the customer; these costs are constant and therefore irrelevant for our purposes. If there is a difference in the repair costs it will be integrated in the costs for passing defective items. For each of the above mentioned factors we must determine what kind of costs are appropriate: costs per item or costs per lot or both.

a) Costs for inspecting a lot

The costs of inspecting a lot are not strictly proportional to the sample size n. Frequently a major part of the inspection costs is fixed. They are based on preparing a machine which is used to inspect the items. Consider for instance the life testing or testing of the strength of a material. Fixed costs are incurred indirectly even when a quick glance at an item is sufficient to determine whether it is defective or not. It is economically significant whether a received lot from a supplier or from a population process within the factory is inspected or not; in the latter case the lot can pass directly to the production process, whereas in the former case there will usually be a delay which causes inventory holding costs. Some of the marginal inspection costs are difficult if not impossible to determine. A quality control department has a restricted capacity for contorlling incoming lots, i.e. there is a certain manpower capacity and only a certain number of machines are available. How to assign costs for using these capacities is not an easy task. Costs in particular cannor be proportionalized to obtain costs per item since the sample size n is not known a priori. Hence, there remain only relatively few propertional cost factors such as material used, electricity and the loss of the item in the case of destructive testing.

It might be argued that fixed sampling costs are irrelevant since they have no effect on the optimal sample size n or on the acceptance number c. But the decision whether to take a sample at all does highly depend on the magnitude of the fixed sampling costs. Furthermore the fixed sampling costs are relevent in deciding whether a single, double, multistage or a sequential sampling plan is optimal. In fact it is rather inappropriate to investigate single, double or multistage sampling plans based on prior distribution and costs without assuming fixed sampling costs, since it is obvious that in the case that there are no fixed sampling costs, a sequential sampling plan is optimal. In the following model we shall therefore consider the situation in which the sampling costs have the form

$$a_0 + a_1 n$$

where a_0 represents the fixed costs and a_1 the cost per item.

b) Costs for accepting a lot with defective items

The economic consequences of undetected defective items are twofold. First let us consider the consequences of defective items in incoming lots or lots from a first production stage. The defective items in accepted lots can cause damages which might be measurable in economic terms. If the items under consideration are to be used as parts in an assembly operation, as is usually the case for incoming lots, the loss by accepting a defective item may consist of costs for identifying and handling the defective item, plus costs of disassembling and assembling the final product. More significant might be the economic consequences of interrupting the production process, which can be very expensive.

Secondly let us consider what happens when defective products are shipped out. The loss by accepting defective final products and shipping them to customers might involve guarantee costs such as service and replacement costs and penalty costs for not fulfilling a contract. But these factors are often not the most critical ones. Even more significant can be the loss of customer goodwill. Goodwill refers to the potential loss of future business from customers and acquaintances as a result of the inability of the firm to satisfy the customers' expectations regarding the quality of the product. Loss of customer cofidence caused by the poor quality of a lot can have serious consequences not only on the future demand of the single product but also on the success of the whole company. Manufacturers are well aware of the value of quality reputation. Some catastrophic failures now and then have dramatized this value. An example of a Japanese manufacturer whose share of market dropped from 70% to 10% after an error which resulted in the death of some babies is mentioned by Juran (1972).

The loss of goodwill is unfortunately very difficult to measure in economic terms. The level of lot quality that is provided is often dependent on qualitative factors, such as the marketing objectives of the firm. Opportunity costs for the loss of goodwill cannot be considered strictly proportional to the number of defecives in a lot, as most other costs can be, at least approximately. In fact, the loss of goodwill increases more rapidly than proportionally to the outgoing quality level. This is due to the snowball effect of loss of goodwill. A case in point: if less than 1% of a particular car have a defective vital part, it might not affect sales too much; on the other hand, if the number of defective cars increases to over 10%, then the drop in demand for the car in question will be drastic. This means that loss of goodwill given 10% defectives is not simply 10 times that of 1% defectives. Consequently, it is by no means easy to assign costs per item for accepting defective material. We will nevertheless assign costs per defective item g as most authors do in order to study the implications of such simplifications in the final section.

c) Costs of rejecting a lot

The costs of rejecting a lot depend on the economic consequences of the rejection. Rejected lots may be sorted, reprocessed, scrapped, used for other purposes, sold at a reduced price or returned to the supplier. If a factory receives a product from a supplier it simply might return the rejected lots. Returning a lot could interrupt the production process if there is no safety stock. On the other hand, holding a safety stock for these situations results in higher inventory holding costs. Thus costs per lot are incurred in any case for returning a lot.

A lot which is produced within the company and rejected by the quality control department before it is shipped out is usually sorted. Besides the costs for sorting and reworking per item there are costs per lot caused by the delay in delivery time, which might have to be introduced as opportunity costs. These costs are again due to a loss of customer goodwill, by which we mean the aversion of a customer to trade with a company which has extreme delays in delivery time. In most cases it seems therefore to be more appropriate to assign costs per lot than only costs per item. In the following model we will assume a fixed cost b_0 of rejecting a lot plus a cost b_1 per item for sorting.

The actual values of all the above mentioned costs are difficult, if not impossible, to determine in practice. All the suggested approaches of measuring the costs result in average rather than marginal costs. Moreover, there is no basis for the measurement of penalty costs for loss of goodwill in accounting methodology. The use of costs in quality control models has thus not been adopted by most practitioners.

Hald (1981) has argued that the problem of determining the above me tioned costs might be reduced to estimating the two quotients $k_s = a_1/g$ d $k_r = (b_1+b_0/N)/g$ where N is the lot size if we restrict ourselves to a single sampling plan. The value k_r is denoted as the breakeven point. But this does not solve the general problem. What are the actual values of k_s and k_r? Some authors assume for convenience that $k_r = k_s$ and set k_r as being equal to the average incoming quality level (AIQL), Pfanzagl (1963).

In order to clarify the problem involved in the approach based on prior distributions and costs, we have to strictly distinguish between the classical objective of quality control, where a process is assumed to be in control, and the method of screening a production which is not in control. The classical "acceptance quality control system that was developed encompassed the concept of protecting the consumer from getting unacceptably defective material, and encouraging the producer in the use of process quality control", Dodge (1969). The objective of acceptance quality control is described by Schilling (1982): "Individual sampling plans are used to protect against irregular degradation of levels of quality in submitted lots below that considered permissible by the consumer. A good sampling plan will also protect the producer in the sense that lots produced at permissible levels of quality will have a good chance to be accepted by the plan. In no sense, however, is it possible to inspect quality into the product."

In contrast to this classical objective, Hald (1960) considers a situation where separating bad from good lots is the objective: "Suppose, for example, that two manufacturing processes are available, a cheap one with a poor distribution and a more expensive one with a better prior distribution. The problem is whether to use the cheap manufacturing process and a relatively large amount of inspection or the expensive process and a smaller amount of inspection. This may be solved by determining the optimum inspection procedure for each of the two processes and afterwards selecting the process having minimum total costs. Similar considerations may be useful for a customer who has to choose between several suppliers, each supplier having his own market price and a corresponding prior distribution". The objective here of sampling inspection is obviously to inspect quality into the product. But there are good reasons to believe that an overall cost evaluation would yield to the conclusion that it is more economical in the long run to keep the production process in control to meet customers' specifications than producing or buying unacceptable quality and screening it afterwards.

If the production process is in control, and this is a basic requirement for instance to apply normal inspection in MIL-STD-105D, then rejecting a lot should be a rather seldom case, say less than 5%. If the number of rejected lots becomes too high, action has to be taken on the producer's side to imrpove lot quality. If we take into consideration this requirement for a reasonable sampling plan, we have to place AIQL $\ll k_r$ in order to obtain a rejection rate of less than 5%. Before going into detail we will describe the stochastics of our model.

2.2 The stochastics

Let us examine a production process which is in control. Lots of size N are formed from this process. The single items in the lot can be classified as defective or non-defective through inspection by attributes. The number X of defectives in the lot is assumed to be a random variable which varies independently from lot to lot, but its distribution remains unchanged over time. Let $\phi_N(X)$ be the prior probability that a lot of size N contains X defectives. The incoming lots have to be judged according to whether they are acceptable or non-acceptable before they are supplied to a production process or to a customer. Several samples of size n_j $j=1,2,\ldots$ can be taken from the lot in order to estimate the lot percentage defectives. For every sample of size $n_j > 0$ sampling costs

$$a(n_j) = \begin{cases} 0 & n_j = 0 \\ a_0 + a_1 \cdot n_j & n_j > 0 \end{cases}$$

are charged.

If we have taken i samples of sizes $n_1, n_2, \ldots, n_j$ and have observed $x_1, x_2, \ldots, x_i$ defectives, then the posterior probability $q(x_{i+1}|n_1, x_1, \ldots, n_i, x_i, n_{i+1})$ of the number of defectives, x_{i+1}, in a further random sample of size n_{i+1}, is assumed to depend upon the history $(n_1, x_1, \ldots, n_i, x_i)$ only through

$$\bar{n}_i = \sum_{j=1}^{i} n_j \text{ and } y_i = \sum_{j=1}^{i} x_j,$$

i.e. $q(x_{i+1}|n_1, x_1, \ldots, n_i, x_i, n_{i+1}) \equiv q(x_{i+1}|\bar{n}_i, y_i, n_{i+1})$ for all $0 \le x_{i+1} \le n_{i+1}$. This holds for hypergeometric sampling. After each sample there are three possible decisions which can be taken: accept the lot, reject the lot, or take a further sample. If the lot is accepted, then, in addition to the sampling costs of former samples $n_1, \ldots n_i$, costs of acceptance

$$g(\bar{n}_i) \cdot E(\bar{n}_i, y_i)$$

are incurred, where $g(\bar{n}_i)$ is a nonincreasing and nonnegative function of the cumulative samples size $\bar{n}_i$ and $E[p|\bar{n}_i, y_i]$ is the posterior expected value of the fraction of defectives in the non-inspected part of the lot, given $\bar{n}_i, y_i$ and a prior distribution $\phi_N(X)$. A rejection of the lot results in the following costs which are additional to the sampling costs of former samples:

$$b(\bar{n}_i)$$

where $b(\bar{n}_i)$ is a nonincreasing and nonnegative function of the cumulative sample size $\bar{n}_i$.

We shall consider a multistage decision problem where at every stage a decision of accepting rejecting or sampling has to be made. Our objective is to minimize the total expected costs for handling a lot. To be more specific we shall consider two models. In the first model, which we will refer to as Model A, we will assume that every nonconforming item found in the sample will be replaced by conforming items, and that the cost of this replacement process is absorbed into the overall manufacturing cost. Furthermore, it will be assumed that rejected lots will be screened and any nonconforming item found will be replaced with conforming items. Hence, the costs for such a guarantee and the costs for screening are

$$g \cdot (N-\bar{n}_i) \cdot E[p|\bar{n}_i, y_i]$$

and

$$b_1 \cdot (N-\bar{n}_i)$$

respectively, where g and b_1 are costs per item. In the Model B we assume nonconforming items found in the sample will not be replaced and if the lot is rejected a fixed cost is charged. Hence

$$g \cdot N \cdot E[p|\bar{n}_i, y_i]$$

and

b_0

are the respective costs. It turns out that Model A yields much higher sample sizes than Model B. In fact Model A can be used as a replacement model to find optimal screening procedures. Model A is therefore appropriate for a manufacturer who wants to screen his own production. For a customer, however, who gets shipments from a supplier, Model B is more appropriate, since a customer usually will neither replace noconforming items in the sample nor screen a rejected lot, but he will send it back to the supplier. For the mathematical treatment of the optimization problem, we shall consider a dynamic programming approach.

Let $V^*(\bar{n},y)$ be the optimal costs in state $(\bar{n},y)$ where $\bar{n}$ is the cumulative sample size and y is the number of defective items found in the sample. $AC^*(\bar{n},y)$ denotes the optimal action (acceptance, rejection or sampling) in state $(\bar{n},y)$.

Furthermore, we have to determine the distribution of the number of defectives, x, in a further sample of size m, taken from the noninspected part of the lot, given the outcome $(\bar{n},y)$. This distribution will be denoted as $q_N(x|\bar{n},y,m)$. Then $V^*(\cdot,\cdot)$ satisfies the functional equation.

$$V(\bar{n},y) = \min\{g(\bar{n})E[p|\bar{n},y],b(\bar{n}),\ f(m,\bar{n},y)\}$$

where

$$f(m,\bar{n},y) = \min_m\{a(m) + \sum_{x=0}^{m} q(x|\bar{n},y,m)\ V(\bar{n}+m,y+x)\}$$

$$AC(\bar{n},y) = \begin{cases} \text{acceptance} & V(\bar{n},y) = q(\bar{n})E(\bar{n},y) \\ \text{rejection} & V(\bar{n},y) = b(\bar{n}) \\ \quad m & V(\bar{n},y) = \min\limits_m\{f(m,\bar{n},y)\} \end{cases}$$

The optimal V* and AC* shall be computed by value iteration applying

Bellman's (1960) principle of optimality. In the dynamic programming formulation we make use of the Markovian property of the sampling process. At any stage the only concern is the cumulative sample size $\bar{n}$ and the number of defectives y and not in what stage the defectives may have occurred. The probability of having x defectives in a further sample of size m is not influenced by the individual results of the previous samples. Hence it follows from the main theorem of dynamic optimization that for every cumulative sample size $\bar{n}$ and number of defectives y one can easily calculate an optimal decision $AC^*(\bar{n},y)$ by value iteration, which might be acceptance, rejection or a further sample of size m. But we are of course interested in the optimality of multiple (n,c) plans, i.e. given $\bar{n}$ there exist acceptance and rejection numbers c^L and c^U depending on $\bar{n}$ such that the optimal decision IV*(n,y) is

$$AC(\bar{n},y) = \begin{cases} \text{accept the lot for } y \le c^L(\bar{n}) \\ \text{reject the lot for } y \ge c^U(\bar{n}) \\ \text{take a further sample} \\ \text{of size } m(\bar{n},y) \text{ for } c^L(\bar{n}) < y < c^U(\bar{n}) \end{cases}$$

The assumptions which have to be made such that a multiple (n,c) plan is optimal are the same as in the single sampling case. Before discussing these assumptions let us first consider the features of a multiple (n,c)-sampling plan.

If the sampling procedure will lead to acceptance or rejection of a lot after at most ℓ samples due to $c_\ell^L=c_\ell^U$, we shall refer to an ℓ-stage sampling plan. Note that these multistage (n,c) sampling plans include: rejection or acceptance of the lot without sampling: $\ell=0$, single sampling plans: $\ell=1$ (Hald 1960), double sampling plans: $\ell=2$ (Pfanzagl 1963), i-stage sampling plans: $\ell=i$, sequential sampling plans: $\ell=N$ (Pfanzagl and Schüler 1970). Since the number of possible stages of a sampling plan can be assigned a priori (as Pfanzagl and Hald do) or can be the result of an optimization procedure (as will be provided in this paper) we will distinguish between an optimal i-stage sampling plan and an overall optimal sampling plan. An i-stage sampling plan is a sampling plan where it is assumed that exactly i samples can be taken. It is called an optimal i-stage sampling plan if there is no better sampling plan with i or less number of samples. With this notation Pfanzagl (1963) investigated an optimal 1-stage and 2-stage sampling plans. An overall optimal plan specifies an optimal number i* of samples which can be taken. Note that there exist only optimal i-stage sampling plans with $i < i^*$, and an optimal i^*-stage sampling plan is an overall optimal plan.

2.3 Optimality of multiple (n,c) sampling plans

The optimality of multiple (n,c) sampling plans is based on the following two assumptions. Let us refer to

$$F(x|\bar{n},y,m) = \sum_{j=0}^{x} q(j|\bar{n},y,m)$$

as the posterior c.d.f. of the number of defectives in a sample of size m given the state $(\bar{n},y)$. In order to provide a feasible solution within the class of (n,c)-sampling plans, the model must possess the following properties:

(i) If it is optimal to accept the lot given y defectives in a sample of of size $\bar{n}$, then it is optimal to accept the lot if there are y-1 defectives in a sample of size $\bar{n}$.

(ii) If it is optimal to reject a lot given $(\bar{n},y)$ then it is also optimal to reject a lot given $(\bar{n},y+1)$.

These two properties hold if the following two assumptions are fulfilled:

Assumption A1 For all $(\bar{n},y) \in S$, $y < N$, $0 \le m \le N-\bar{n}$ and $0 \le x < m$ the inequality

$$F(x|\bar{n},y+1,m) \le F(x|\bar{n},y,m)$$

holds.

Assumption A2

$$F(x|\bar{n}+1,y,m) \leq F(x|\bar{n},y,m)$$

The first assumption requires that the probability of finding more than x defective items in a further sample of size m given $(\bar{n},y)$ increases with y, the number of defectives in the cumulative sample of size $\bar{n}$. This is a reasonable assumption and is met by many prior distributions. For it implies that if a sample is of good quality a further sample is also likely to be of good quality, or, if a sample is of poor quality, a further sample is similarly likely to be poor. Notice that from Assumption (A1) it follows that

$$E(p|\bar{n},y+1) \geq E(\bar{n},y).$$

We only have to set $m = N-\bar{n}$. The second assumption requires that the probability of finding more than x defective items in a further sample of size m given y defectives in $\bar{n}$ is higher than that of finding more than x defective items in a sample of size m given y defectives in $\bar{n}+1$.

Theorem Assume the assumptions (A1) and (A2) hold, then there exists an optimal multistage (n,c) sampling plan.

The formal proof of that theorem is given in Schneider and Waldmann (1982). It is an extension of the well-known result in the single sampling case, where A1 and A2 have to be assumed too in order to prove an (n,c) plan to be optimal. The assumptions hold for various distributions such as binomial, mixed binomial, Polya and uniform, as shown by Case and Keats (1982). The assumptions are not fulfilled for the hypergeometric distribution (Case and Keats (1982)). The binomial distribution is not an appropriate distributinon in practice since there can be no inference drawn about the quality of the rest of the lot based upon sample results.

The uniform distribution is very unrealistic since it is assumed that every possible lot fraction defective is equally likely. These two authors showed that acceptance sampling works well with the Polya and the mixed binomial distribution. Although the figures presented by Case and Keats are impressive, the gain through sampling should be expressed in economical terms. This is what we will attempt to show.

2.4 The Polya distribution

The Polya p.d.f. is

$$\phi_N(X,\alpha,\beta) = \binom{N}{X} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+x)\Gamma(\beta+N-x)}{\Gamma(\alpha+\beta+N)}$$

where N is the lot size. The average percent defectives p in a lot is (Pfanzagl (1963)) $E[p|0,0] = \frac{\alpha}{\alpha+\beta} = \text{AIQL}$.

One of the reasons this distribution is considered in many papers as a distribution for the number of defectives in a lot is mainly due to the fact that this distribution is very easy to handle mathematically. For this type of prior, for instance, the distribution of the number of defectives in a sample of size n is (if hypergeometric sampling is used)

$$\phi_n(x,\alpha,\beta) = \binom{n}{x} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+x)\Gamma(\beta+n-x)}{\Gamma(\alpha+\beta+n)}.$$

Thus it is again a Polya distribution. This property is referred to as "reproducibility to hypergeometric sampling". The posterior expected value of the fraction of defectives in a lot is then

$$E[p|n,x] = \frac{\alpha+x}{\alpha+\beta+n}$$

After relatively simple calculation we obtain for the expected probability that in a sample of size n there are x defectives, given that sampling has already produced the pair $(\bar{n},y)$

$$q(x|\bar{n},y,m) = \frac{\Gamma(\alpha+\beta+\bar{n})}{\Gamma(\beta+\bar{n}-y)\Gamma(\alpha+y)} \binom{m}{x} \frac{\Gamma(\alpha+y+x)\Gamma(\beta+\bar{n}+m-y-x)}{\Gamma(\alpha+\beta+\bar{n}+m)}$$

It can easily be seen that Assumptions (A1) and (A2) are fulfilled.

2.5 Mixed binomial distribution

The mixed binomial is a realistic prior distribution, except when it consists of only one component; in this particular case, it reverts to the binomial.

The mixed binomial is applicable, for example, when ℓ different machines supply parts, with source i furnishing a proportion w_i, produced at process fraction defective p_i.

The mixed binomial with two components is very appropriate for modelling a process in control with outliers. We consider the following situation. Most often, say in 95% of all cases, the process is in control and produces at process fraction defective p_i. There is about a 5% chance, however, that the process will go out of control, and lots of higher percent defectives will be produced (p_2), which are usually denoted as outlier lots.

The mixed binomial distribution is given as

$$\phi_N(x) = \sum_{i=1}^{\ell} w_i \binom{N}{x} p_i^{\,x} \, (1-p_i)^{N-x}$$

$$0 \le p_i \le 1$$

$$\sum_{i=1}^{\ell} w_i = 1$$

$$x = 0,1,\ldots,N$$

The average process fraction defectives is

$$\text{AIQL} = \sum_{i=1}^{\ell} w_i \, p_i$$

Since the mixed binomial is reproducible to hypergeometric sampling, the probability of the number of defectives in a sample of size $\bar{n}$ is

$$\phi_{\bar{n}}(x) = \sum_{i=1}^{\ell} w_i \binom{\bar{n}}{x} p_i^{\,x}(1-p_i)^{\bar{n}-x} \qquad x = 0,1,\ldots,\bar{n}$$

Furthermore, given that sampling has produced the pair $(\bar{n},y)$, i.e. in the cumulative sample of size $\bar{n}$, y defectives are found, the probability of having $x=0,1,\ldots,m$ defectives in a further sample of size m is

$$q(x|\bar{n},y,m) \sum w_i' \binom{m}{x} p_i^{\,x}(1-p_i)^{m-x}$$

where

$$w_i' = w_i \frac{p_i^{\,y}(1-p_i)^{\bar{n}-y}}{\sum w_i p_i^{\,y}(1-p_i)^{\bar{n}-y}}$$

w_i' is denoted as the posterior probability. Given the probability w_i and taking a sample of size $\bar{n}$ with y defectives in it, the expected fraction of defectives in the noninspected part of the lot is

$$E(\bar{n},y) = \frac{\sum_{i=1}^{\ell} w_i p_i^{\,y+1}(1-p_i)^{\bar{n}-y}}{\sum_{i=1}^{\ell} w_i p_i^{\,y}(1-p_i)^{\bar{n}-y}}$$

2.6 The inefficiency of multiple (n,c)-plans

In the following example we consider Model A in which the cost functions are

$$a(n) = \alpha_0 + a_1 \cdot n \qquad n>0$$
$$g(\bar{n}) = g \cdot (N-\bar{n})$$
$$b(\bar{n}) = b_1 \cdot (N-\bar{n})$$

The prior distributions are taken from Pfanzagl (1963). In his paper single and double sampling plans were investigated under the assumption that there are neither fixed costs for sampling nor sorting. The costs were $a_0=0$, $b_0=0$, and $k_r=k_s$ = AIQL = 0.02. Since $a_0=0$ a sequential sampling plan would be optimal. We have therefore put $a_0=1$, the size of the unit guarantee cost g. We notice also that the breakeven point k_r is much too small. But since the maximal gain through sampling is obtained for k_r = AIQL, the efficiency of multiple sampling plans will even decrease with increasing k_r. Some examples of multiple sampling plans are given in Table 1. We see that multiple (n,c)-sampling plans do not offer nearly the same savings as single sampling plans do. The savings of single sampling plans lie between 15% and 31%, whereas the multiple sampling plan compared with single sampling plans allows only a further reduction in costs of 1%.

Let us examine the situation where the lot size is N=800. If $\alpha=2$ and $\beta=98$ a 1-stage plan is optimal; if $\alpha=1$ and $\beta=49$ a double sampling plan is optimal. If, however, the lot size is N=400, then a single sampling plan is optimal for $\alpha=1$ and $\beta=49$.

TABLE 1 Optimal multiple sampling plans for Model A

N	$\alpha+\beta$	α	n	c	V	$n_1c_1^Lc_1^U$	y_1	$n_2c_2^L$	V	$n_1c_1^Lc_1^U$	y_1	$n_2c_2^Lc_2^U$	y_2	$n_3c_3^L$	V	i*	Inevitable costs
400	50	1	73	1	7.14											1	4.9135
	100	2	78	1	7.84											1	5.6084
800	50	1	120	2	12.82	93 1 3	2	137 4	12.77							2	9.9958
	100	2	128	2	14.28											1	11.4554
1600	50	1	173	3	23.82	113 1 4	2 3	211 6 277 7	23.60	114 1 4	2 3	181 5 7 270 7	6	100 7	23.59	3	20.7668
	100	2	224	4	26.85	162 2 5	3 4	217 7 228 7	26.74							2	23.1632
3000	50	1	227	4	42.68	134 1 5	2 3 4	243 7 257 7 261 7	42.22	137 1 5	2 3 4	139 4 7 254 7 258 7	5 6	118 7 121 7	42.21	3	37.9690
	100	2	324	6	48.44	187 2 6	3 4 5	200 7 207 7 210 7	48.23							2	43.6582
			single sampling plan			double sampling plan				3-stage sampling plan							

*maximal stages of an optimal multistage sampling plan, a_0= 1, Polya distribution

The 3-stage sampling plan for N=1600 reads as follows: Take a sample of size n=114. If the number of defectives x_1 in the sample is $x_1=1$ or less the lot has to be accepted. If $x_1=4$ or more, reject the lot. If $x_1=2$ take a further sample of size $n_2=181$. If the number of defectives $y_2=x_1+x_2$ in the cumulative sample of size $n_1+n_2=295$, is $y_2=5$ or less, accept the lot; if $y_2=7$ or more reject the lot and if $y_2=6$ take a further sample of size 100. The lot is accepted if the number of defectives $y_3=x_1+x_2+x_3$ in the sample of size $\bar{n}_3=n_1+n_2+n_3$ is less than or equal to 7; otherwise the lot is rejected. If the number of defectives x_1 in the first sample n_1 is 3, then another sample of size $n_2=270$ is taken. In this case the sampling procedure stops after two stages. These multiple sampling plans look rather complicated and the small savings offer no incentive to apply them. In fact, if we take into account that in practice two thirds of the sampling costs are fixed, then a_0 will be greater than 1 and none of the multiple sampling plans will be optimal. This illustrates that with the assumptions of our model and in particular fixed sampling costs multiple sampling plans are not optimal.

2.7 The efficiency of single sampling plans

In Table 2 we reconsider the single sampling plans for the examples given in Table 1. As seen in Table 2 for lot size N=400, $\alpha=1$, $\beta=49$, k_r=AIQL, the savings of a single sampling plan are 31%. But where do these savings come from? This is easily demonstrated when evaluating the percentage of rejected lots. In all cases about 40% of the lots are rejected. Is it therefore realistic to state that k_r=AIQL? It is certainly not common in practice to reject 40% of the lots as we have discussed above. In order to obtain a realistic rejection rate of less than 5% the cost parameters have to be such that k_r>>AIQL. We should remember that here for purposes of investigation we have selected a certain prior distribution, which implies that AIQL is given; thus we are only able to vary the breakeven point such that a realistic rejection rate can be achieved. In practice, however, the inverse would be true: the breakeven point is given and we have to ensure that the prior distribution is concentrated far below this point.

TABLE 2 Optimal sampling plans for the Polya distribution

				Model A					Model B				
α	β	N	C^0	n	c	C*	Δ%	rej %	n	c	C*	Δ%	rej %
2	98	400	8	78	1	6.8	15	41	22	0	7.5	7	33
		800	16	128	2	13.3	17	41	31	0	14.4	9	42
		1600	32	224	4	25.9	19	40	76	1	27.8	13	40
		3000	60	324	6	47.4	21	41	126	2	50.4	16	41
1	49	400	8	73	1	6.1	31	36	27	0	6.7	16	36
		800	16	120	2	11.8	26	36	34	0	12.9	20	41
		1600	32	173	3	22.8	29	37	78	1	24.4	24	38
		3000	60	227	4	41.7	30	37	126	2	43.9	27	37

k_r=0.02, AIQ=0.02, C^0: cost without inspection, C*: optimal costs,
Δ%=$(C^0$-C*$)/C^0$, rej %: percentage of rejected lots

The effect of moving the breakeven point k_r to the right causes a drastic drop in savings which result from sampling. It seems to be a trivial result that high savings of sampling plans are due to a high percentage of rejections. But the consequences are that even single sampling plans turn out to be less cost efficient than Hald (1960) and Pganzagl (1963) stated, concerning the model considered in this paper. In order to demonstrate this we have selected the example N=800, α=1, β=49. Table 3 shows the savings and rejections for different breakeven points k_r. The fixed sampling costs were placed at zero.

TABLE 3 Optimal gain and rejection rates of single sampling plans

	Model A		Model B	
k_r	Δ%	rej %	Δ%	rej %
0.02	26	36	20	41
0.025	16	26	11	23
0.030	9	18	5	15
0.035	4	11	1	8
0.040	1	6	0	0

$\Delta\% = (C^0 - C^*)/C^0$, rej %: percentage of rejected lots

Low rejection rates imply low savings. By introducing even small fixed sampling costs here, we realize that acceptance without rejection would be optimal. This holds of course only for the Polya distribution.

The mixed binomial distribution with two components allows us to construct situations where single sampling plans are very efficient and have low rejection rates. Let us consider for example a distribution where 95% of the lots have an average percentage defective $p_1=0.01$ and 5% outliers with average $p_2=0.27$; thus AIQL=0.02 and the variance is $\sigma^2=0.0019$. We set $k_r=0.04$ to separate the 5% outliers from the 95% good lots. The results for the two lot sizes N=400 and N=1600 are presented in Table 4.

TABLE 4 Mixed binomial prior distribution

N	n	c	C^*_m	Δ%	rej %	C^*_p	$\frac{C^*_p - C^*_m}{C^*_m}$ 100
400	18	1	5.76	28	6	6.21	8
1600	32	2	20.18	37	5	22.02	9

$\Delta\% = (C^0 - C^*)/C^0$, rej %: percentage of rejected lots

C^*_m: optimal costs, C^0: costs of acceptance without inspection

C^*_p: costs of a Polya plan, Model B

Comparing the costs C_m^* with the costs C^0 which amount to 18 for N=400 and 9 for N=1600, it is obvious that sampling is optimal and the gain of sampling is very high even if there are additional fixed sampling costs. This is not the case for multiple sampling plans. Their efficiency is less than 1% for all mixed binomial distributions which we have considered in our study.

We have also considered a Polya distribution with mean AIQL = 0.02 and variance σ^2=0.0019. The appropriate parameters are α=0.18631 and β=9.12942.

TABLE 5 Polya prior distribution

N	n	C	C_p^*	Δ%	rej %	C_m^*	$\frac{C_m^*-C_p^*}{C_p^*}$ 100
400	12	0	5.83	27	15	6.08	4
1600	35	1	20.58	36	15	21.70	5

C_p^*: optimal costs, C_m^*: costs of a mixed binomial plan, model B

Table 5 shows that the savings assuming a Polya distribution are also high but they are linked with high rejection rates (15%). If we try to lower the high rejection rates we would once again obtain a low efficiency of single sampling plans.

In the last columns of Tables 4 and 5 we have given the costs in case we used the wrong distribution to calculate a single sampling plan. We see that in our specific case the losses are lower if we apply a mixed binomial plan.

The only model which offers any significant savings through sampling seems to be the mixed binomial model where a high percentage of the lots is good and a low percentage of the lots, usually denoted as outliers, is very poor. In all other cases we may only gain by rejecting a high percentage of the lots. But this implies a very poor prior distribution which few customers would accept. Actions on the supplier's part to improve lot quality would be the reaction which would be most economically sensible and thus usually carried out in practice.

3. An empirical study of single sampling plans

In this section we shall present an empirical study of single sampling plans with respect to their efficiency and robustness. Seven empirical distributions which were investigated in a Ph.D. thesis by Fizner (1980) form the basis of this study. In columns 2 and 3 of Table 6 the average percentage of defectives in a lot and the variance are given, where the entries of column 3 have to be multiplied by 10^{-6}. It should be noted that the total number of defectives in a lot is rarely known. In most practical situations only the number of defectives in a sample is available. But even in a situation where we know the frequency of the number of defectives in a batch we have the problem of assigning an appropriate prior distribution. Two models to describe the data are considered: the Polya and the mixed binomial distribution. Both are based on different assumptions concerning the production process. If we know nothing about this process as a customer, then we have no idea which of the two distributions we should choose. Hence it is of great interest whether the costs are sensitive with respect to the prior distribution. Firstly, the two distributions were fitted to the empirical data as described below.

The Polya model

Given the number of defectives x_i and the frequency r_i ($\Sigma r_i = r$) we use

$$\hat{\alpha} = (1-\hat{p})\left(\frac{\hat{p}^2}{\hat{\sigma}^2} + 1\right) - 1$$

$$\hat{\beta} = \hat{p}\left(\frac{(1-\hat{p})^2}{\hat{\sigma}^2} + 1\right) - 1$$

as estimates of α and β where

$$\hat{p} = \frac{1}{N}\left(\frac{1}{r}\,\Sigma\, r_i X_i\right)$$

$$\hat{\sigma}^2 = \frac{rN-1}{rN^2(N-1)}\, S_x^2 - \frac{1}{(N-1)}\,\hat{p}(1-\hat{p})$$

$$S_x^2 = \frac{1}{r-1}\sum_{i=1}^{k} (x_i - N\hat{p})^2$$

$\hat{p}$ and $\hat{\sigma}^2$ are unbiased estimates of E[P] and Var[P].

The mixed binomial model with two components

We used the moment estimator for the mixtures of binomial distributions. The factorial moments are defined as follows:

$$V_j = \frac{1}{r}\sum_{i=1}^{r} \frac{x_i(x_i-1)\cdots(x_i-j+1)}{(N-1)\cdots(N-j+1)}$$

$j=1,2,3$

Explicit formulae can be obtained (see Johnson and Kotz (1969)), for the estimates $\hat{w}_1, \hat{w}_2, \hat{p}_1 \hat{p}_2$

$$\hat{p}_1 = \frac{1}{2}A - \frac{1}{2}(A^2 - 4AV_1 + 4V_2)^{1/2}$$

$$\hat{p}_2 = \frac{1}{2}A + \frac{1}{2}(A^2 - 4AV_1 + 4V_2)^{1/2}$$

$$A = (V_3 - V_1V_2)/(V_2 - V_1^2)$$

$$\hat{w}_1 = (V_1 - \hat{p}_1)/(\hat{p}_1 - \hat{p}_2), \hat{w}_2 = 1 - \hat{w}_1$$

The estimates $\hat{\alpha}, \hat{\beta}, \hat{w}_1, \hat{w}_2, \hat{p}_1, \hat{p}_2$ are presented in Table 6.

TABLE 6 Parameters of empirical distributions

	$\hat{p}$	$\hat{\sigma}^2$	$\frac{\sigma^*}{p^*}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{w}_1$	$\hat{p}_1$	$\hat{p}_2$
D1	0.01608	115.5	0.67	2.187	133.797	0.85144	0.0116	0.04176
D2	0.01188	29.9	0.45	4.641	386.221	0.79801	0.0091	0.02273
D3	0.00290	5.9	0.83	1.416	486.780	0.86974	0.00196	0.00917
D4	0.00149	1.6	0.83	1.678	1126.089	0.86646	0.00104	0.00440
D5	0.00142	3.2	1.25	0.642	448.769	0.95451	0.00104	0.00956
D6	0.00173	1.1	0.63	2.592	1495.614	0.87783	0.00133	0.00460
D7	0.00581	95.3	1.67	0.347	59.280	0.85434	0.00180	0.02932

The optimal single sampling plans were first determined for k_r=AIQL, since this parameter combination involved the largest cost deviation between a Polya and a mixed binomial distribution. We shall now mention a few interesting results obtainable from Tables 7 and 8. These tables give the optimal sampling plans and costs, the rejection rates and the costs for a mixed binomial plan if the distribution is Polya and vice versa. The results reveal that the costs are not very sensitive with respect to variations of the prior unless the coefficient of variation σ/E[P] becomes larger than 1. For example, the optimal cost (C_p^*) for D7 in Table 7 is 31.3 and 26.6 (C_m^*) in Table 8. But these are the only distributions where sampling seems to be effective.

TABLE 7 Optimal single sampling plans for Model B

	Prior: Polya, plan: Polya						Prior: Polya plan: mixed binomial			
	n	c	C_p^*	C_0	rej%	Δ_0%	n	c	C_m^*	Δ_0^*%
D1	342	5	130.8	160	41	18	320	7	135.3	3
D2	293	3	108.6	118	43	8	432	6	109.9	1
D3	498	1	24.3	29	37	16	538	2	24.8	2
D4	416	0	13.2	15	41	12	635	1	13.4	2
D5	498	0	10.2	14	38	27	428	1	11.2	10
D6	374	0	15.9	17	44	7	269	0	15.9	0
D7	396	2	31.3	58	27	46	340	3	32.9	5

C_p^*: optimal costs, C^0: costs of no inspection

C_m: costs of the optimal mixed binomial plan, $\Delta_0\% = (C^0-C_p^*)/C^0$

TABLE 8 Optimal sampling plans for Model B

	Prior: mixed binomial, plan: mixed binomial						Prior: mixed binomial, plan: Polya			
	n	c	C_m^*	C_0	rej%	Δ_0%	n	c	C_p^*	Δ_0^*%
D1	320	7	130.6	160	19	18	342	5	135.98	4
D2	432	6	107.7	118	29	8	293	3	109.2	4
D3	538	2	24.4	29	21	16	498	1	25.2	3
D4	635	1	13.4	15	24	11	416	0	13.7	2
D5	428	1	11.7	14	11	16	498	0	12.7	9
D6	269	0	16.2	17	36	5	374	0	16.3	1
D7	340	3	26.6	58	17	54	396	2	27.8	5

C_m^*: optimal costs, C^0: costs of no inspection

C_p: costs of the optimal plan, $\Delta_0\% = (C^0-C_m^*)/C^0$

In order to lower the rejection rate we increased the breakeven point k_r. The results are presented in Table 9.

TABLE 9 Optimal gain and rejection rate for Model B

		Polya			Mixed Binomial		
	k_r	Δ%	rej%	Δ*%	Δ%	rej%	Δ*%
D1	0.028	1	7	1	6	14	1
D2	0.015	1	15	1	3	21	1
D3	0.005	0.3	6	1	3	11	1
D4	0.002	1	14	1	3	13	1
D5	0.003	2	8	1	5	5	1
D6	0.002	2	23	1	2	15	1
D7	0.025	2	3	1	2	12	1

Δ*% gives the loss by applying the wrong distribution model
Δ% gives the gain of sampling
rej % is the percentage of rejected lots

The gain through sampling decreases drastically with increasing breakeven point k_r. But it decreases faster in the Polya model. In the mixed binomial model the percentage of rejected lots is nearly constant and amounts to almost $1-w_1$ as long as k_r lies between p_1 and p_2.

For all the distributions which we considered, sampling does not seem to be very efficient regardless of which distribution model is used. Only distribution D5 offers a gain of 5% along with 5% rejections if the mixed binomial model is assumed. If we add only small fixed sampling costs, for example the size of the variable sampling costs $a_1 \cdot n$, then for almost none of the considered cases would sampling be optimal. To illustrate this, in Table 10 we have given the optimal costs without fixed sampling costs and the variable sampling costs.

TABLE 10 Optimal costs of single sampling plans

	k_r	C_0	C^*_p	C^*_m	C^s_p	C^s_m
D1	0.028	160.8	159.6	151.5	2.97	6.83
D2	0.015	118.8	117.8	115.4	2.36	5.36
D3	0.005	29.0	28.9	28.2	0.60	1.69
D4	0.002	14.9	14.7	14.6	0.21	0.78
D5	0.003	14.2	14.0	13.4	0.77	1.25
D6	0.002	17.3	17.0	16.9	0.32	0.77
D7	0.025	58.1	56.8	57.0	2.28	3.15

C^*_p, C^*_m: optimal costs
C^0: costs for acceptance without sampling
C^s_p, C^s_m: variable sampling costs $a_1 \cdot n$

We have to remember that in our examples the lot sizes are very large (10,000); hence the results hold for cases where the lot size is even more noticeably smaller.

The sensitivity of the optimal costs with respect to the sampling plan used was also under investigation. An evaluation of the costs as a function of the sample size n shows that the costs are not very sensitive to changes of the sample size.

Table 9 gives the cost deviations if the wrong distribution model is used. Costs differ from the cost minimum by less than 1% although the sampling plans are very different. For example, in the case where a Polya distribution is assumed (k_r=0.015, D2), the optimal sampling plan is n=157, c=3; if we assume a mixed binomial distribution the optimal plan is n=357, c=6.

4. Conclusion

This paper has dealt with a sampling model based on prior distribution and costs, which encompasses most of the existing Bayesian models based on costs. We pointed out the difficulties of determining the marginal costs which are needed for the evaluation of the model. Because of the lack of information about these costs we have suggested varying the breakeven point such that a rejection rate is obtained which is considered reasonable in practice; otherwise actions have to be taken to improve lot quality.

The conclusion that may be drawn from the presented study is twofold. Firstly, we will consider the implications if our model is valid. Secondly, since every model is only an approximation to reality, we shall consider some of its assumptions and discuss whether they are fulfilled in practice.

Let us first assume our model is suitable for practical purposes, i.e. we believe there is a prior distribution and piecewise linear costs can be assigned. Taking into consideration the constraint that the percentage of rejected lots is low, then the prior has to be centered to the extreme left of the breakeven point k_r; thus, sampling becomes only efficient in an outlier model, i.e. where most of the lots have a good quality level and some lots have a high percentage of defectives. To be more specific, it does not pay to keep a sampling system running just to detect that about 5% of the lots have a quality slightly higher than the AQL. This would contradict the assumption of linear costs. The only relevant sampling plans have thus a small sample size with a flat operations characteristic. This would support the use of sampling plans as given in the MIL-STD-105D "reduced inspection".

In designing a sampling plan for a new product one does not usually have prior information; thus the use of a sampling plan with a steep OC-curve might be appropriate. If the rejection rates are high, a discontinued phase is advisable which involves stopping shipment of the product while the supplier performs corrective action. If the rejection rate is low it does not pay to sample with a steep OC-curve; hence the inspection should be reduced. But in terms of linear costs, it is not optimal to choose a sampling plan from a table for normal inspection or even tightened inspection such as MIL-STD-105D and apply it regardless of how high or low the rejection rate is.

It should be clear that these results are only valid within the model we have presented. Some assumptions of the model might not be met in practice. The lot quality, for instance, might not be independent from lot to lot. The assumption of a Markov process could then be more appropriate. The objective of a sampling system is then to detect as early as possible whether the lot quality has changed from a good to a poor quality level. A shift in the quality level should of course result in action on the supplier's side to improve the quality rather than continuing sampling in order to screen the poor lots. Many sampling systems are therefore designed like feedback systems.

Another critical point in our model is the assumption of linear costs. Nonlinear guarantee costs may be more suitable in practice if loss of goodwill is taken into consideration, as pointed out earlier. It was shown by Schneider and Werner (1981), for instance, that linear costs are not equivalent to assigning an average outgoing quality limit (AOQL). This means, if an AOQL is required, nonlinear guarantee costs are necessarily involved. Nonlinear guarantee costs could also be the reason that practitioners use sampling plans with a steep OC-curve even if there are only a small number of rejected lots.

The paper sheds new light on the economic consequences of sampling inspection. Case and Keats (1982) stated that "designer and users of these plans should be aware of the implications associated with the selection of this prior distribution". We might add that they should also be aware of the economic assumptions and consequences associated with the selection of a sampling plan.

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